1 OF 1
ADA034878

END
DATE
FILMED
2 -77

JAN 26 1977

A STUDY OF THE METHOD OF

SURFACE FIT APPROXIMATIONS

FOR AIRCRAFT OPTIMAL DESIGN

THESIS

GA/MC/76D-9   Martin L. Marler
                                     Captain   USAF
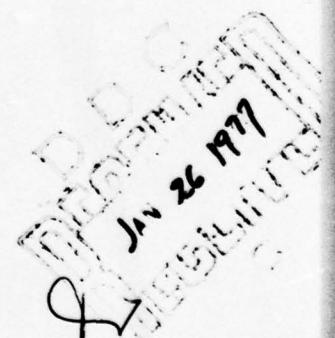
A STUDY OF THE METHOD OF
SURFACE FIT APPROXIMATIONS
FOR AIRCRAFT OPTIMAL DESIGN
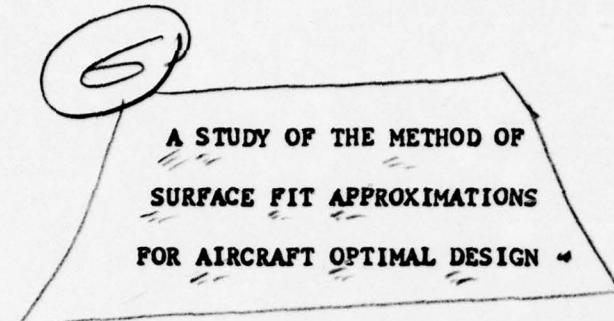
Master's thesis

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

Martin L. Marler, B.S.

Captain   USAF

Graduate Astronautical Engineering

October 1976

72p.

Preface

This study was the first attempt at providing a usable set of computer routines (which employed current concepts for the selection of the design space, second order surface fit approximations, and a conjugate gradient minimization algorithm) to be used by the Air Force Flight Dynamics Laboratory Design Branch in the initial design phase of an aircraft. Several proposals and programs were available but the lack of information as to the use and application of these concepts led to this study. Computer routines that had been working were found not to be compatible with the Wright-Patterson Air Force Base computer system and had to be modified and rewritten. This study attempted to eliminate these problems and to demonstrate the usefulness of the surface fit approximation scheme.

I would like to thank Mr. Jim Parker and Captain Russ Morrison of the Air Force Flight Dynamics Laboratory Design Branch for their assistance in providing me with input information for the mission simulation used in this study.

I would also like to thank Dr. K. S. Nagaraja of the Air Force Flight Dynamics Laboratory Aerodynamics Branch and Major Steve Koob of the Aeromechanical Engineering Department of the Air Force Institute of Technology who provided the thesis topic and valuable aid during the course of this study.

Finally, I would like to express my appreciation to Major Gerald Anderson of the Mechanics Department of the Air Force Institute of Technology who agreed to sponsor this thesis topic.

Captain Martin L. Marler

Contents

List of Figures

List of Tables

Abstract

A study was made to demonstrate the feasibility of using surface
fit approximations in the design analysis of an advanced tactical
fighter. Design variables were selected and a design space was defined
based on a simple latin square method. The take-off-gross-weight (TOGW)
and landing distance (DISTL) for each configuration were determined by
use of a computer program which simulated the required mission. A
regression analysis was performed on this data to obtain a quadratic
surface fit representing TOGW and DISTL as functions of the design
variables. A conjugate gradient technique was employed to find the
minimum TOGW subject to constraints on DISTL within the design space.
This value was compared to the minimum obtained by running all possible
combinations of design variables through the mission simulator. It was
concluded that the surface fit approximation based on the latin square
design space did yield accurate results and required fewer computer
simulations.

A STUDY OF THE METHOD OF

SURFACE FIT APPROXIMATIONS

FOR AIRCRAFT OPTIMAL DESIGN

I. Introduction

The Air Force Flight Dynamics Laboratory (AFFDL) Design Branch
became interested in several proposals concerning the selection of an
airplane engine by optimization of surface fit approximations. The
original studies had been conducted by the Aero Propulsion Laboratory
(APL) at Wright-Patterson Air Force Base. The results of these studies
showed that considerable savings could be made in engineering and com-
puter time by using limited quantities of simulation data (Ref 7:599).
The AFFDL Design Branch was involved in a design scheme similar to those
used in the aforementioned studies. The Design Branch wanted to inves-
tigate the use of surface fit approximations based on the latin square
design selector and integrate these ideas into their design analysis.

In particular, the Design Branch wanted to apply the method to an
aircraft that would be capable of accomplishing an interdiction mission
that might be required of an advanced tactical fighter. The vehicle
was to have a one man crew, carry 5500 lbs of stores, and be capable
of a supersonic range of up to 350 nautical miles.

The purpose of this study was to demonstrate the feasibility of
using surface fit approximations in the design analysis of an advanced
tactical fighter. The design process that was considered consisted of
six major steps. The first step required the selection of independent

1

design variables (engine, airframe, and/or mission) and dependent performance or response functions. The performance functions would be calculated based on the independent design variables. The independent variables selected were the wing loading (airframe variable), aspect ratio (airframe variable), vehicle thrust to weight ratio (engine variable), and the supersonic dash range (mission variable). The dependent performance functions were the take-off-gross weight and the landing distance. The next step was to choose values for the independent variables. This was done by establishing the upper and lower limits on each variable and then dividing that range into an equal number of parts (equal to the number of independent variables). The ranges of the independent variables were chosen so that the performance functions could be adequately represented by quadratic equations in the regions of interest. The third step was to select from all possible combinations of the independent parameters that subset which would still be representative of the complete design space. This reduces the number of data points needed and also reduces the computational time for deriving the performance function values. This was accomplished by a statistical technique called the simple latin square design selector. This technique joins together permutations of the various values of the independent variables in such a way that each variable is stepped through all its values once every "l" data points where "l" is the number of values for each variable. The fourth step was to compute the dependent performance parameters by inputing the selected design variables into a mission simulator and outputing the performance values. The available mission simulator was that used by the AFFDL Design Branch (the

2

Computerized Initial Size Estimate - CISE). This program is used to give initial values of the design parameters at the "zero level" of analysis, i.e., even before a drawing of the configuration is proposed (Ref 4:2). The program performs a weight oriented sizing process to produce the basic physical characteristics of the vehicle. The next step was to perform a regression analysis on the performance data to yield quadratic surface fits representing the performance functions in terms of the independent design variables. Finally, a constrained minimization of the performance function of primary interest was performed and located those values of the independent parameters that yielded the constrained minimum.

The number of independent variables in this study was taken to be four with each variable taking on five values each. The number of variables was limited so that all possible design combinations could be calculated and input into the mission simulation. The minimum design point from the complete set could then be compared to that found by using the surface fit approximations.

II. Design Space Selection

General

The first part of the study was to identify those variables that might be significant in the design of an aircraft given a general mission. A distinction is made between dependent and independent variables. The dependent variables are selected by considering the total design goal. The TOGW is generally used as one dependent variable since it can be calculated from the various other parameters by using a mission simulation. The independent variables are chosen based on the ability of the designer to change these parameters and their impact on the total design objective. These parameters (both dependent and independent) are typically engine variables (bypass ratio, thrust to weight ratio, etc.), airframe variables (wing loading, aspect ratio, leading edge sweep angle, etc.), or mission parameters (range, altitude, loiter ability, etc.).

In this study, it was desirable to have at least one variable from each of the three possible sets of airframe, engine, and mission. The mission parameter selected was the supersonic dash range (RN). It was chosen because it could be varied in the CISE program and is a parameter of interest in the advanced tactical fighter. The engine parameter that was chosen was the thrust to weight ratio (TW), i.e., the ratio of the installed engine thrust to the vehicle weight. Two airframe parameters were selected. The wing loading (WOS) and aspect ratio (AR) were chosen from this group. They represent two characteristics that are available for use in the CISE program.

The next step in the design space selection was to determine a reasonable range for each variable. The ranges were chosen so as to allow a second order polynomial in terms of the independent variables to approximate the dependent parameters (to be discussed in Chapter IV). The ranges in this study were based on previous aircraft of the same type having a comparable mission. The range of each variable was then divided into four equal segments so that all four variables would have five possible values. The possible values of the four parameters were chosen as follows:

(Airframe Variable)	WOS	70	80	90	100	110
(Airframe Variable)	AR	2.5	3.0	3.5	4.0	4.5
(Engine Variable)	TW	.5	.7	.9	1.1	1.3
(Mission Variable)	RN	150	200	250	300	350

where WOS has units lbs/sq ft and RN has units of nautical miles. There are 5^4 possible combinations that made up the complete design space. These 625 design points were input to the CISE program which gave as output 625 TOGW's and DISTL's.

Latin Square Design Selector

The computer time required to run 625 mission simulations was approximately 600 seconds. The need to reduce this time and yet maintain some degree of accuracy in the final optimal design point led to the use of a statistical design selector. The latin square design selector will select out of the entire design space an appropriate subset which yields a good representation of the complete space. The representation is good in the sense that approximating equations for the dependent variables based on the latin square subset will predict

the actual or true value to within five percent. The simple latin square technique uses a method (to be discussed later) which yields a sequence of values for each variable. The sequence is formed by joining together permutations of the various values of the variables in such a way that each variable is stepped through all its values once every "1" data points, where "1" is the number of values for each variable.

The number of data points selected by the latin square technique depends on the number of independent variables. If there are "n" design variables then there will be $(n+1)*(n+2)/2$ terms in the regression equation and the regression routine will require at least this many data points. The simple latin square will generate $(n+1)^2$ design points that have design values of each independent variable equally spaced on the normalized interval $(-1,1)$ (Ref 2:4-5). Accurate fits may not be obtained with this small number of points so the simple latin square technique was modified to generate more data points (this was done to gain more accuracy and at the same time the time for computer simulations of the mission was increased). Table I contains the number of data points required by the modified latin square for a given number of variables.

The simple latin square technique uses field algebras, that are derived from modulo (or residues class) arithmetic using the algebra of integers, to establish "n" matrices (where again "n" is the number of independent design variables) that are used to generate the design points (Ref 6:57). Table II contains the matrices for the four variable case.

Table I
Number of Design Points Required by Latin Square

No. of Variables	No. of Design Points
2	9
3	15
4	45
5	45
6	91
7	91
8	231
9	231
10	231
11	231
12	325
13	325
14	561
15	561
16	561
17	561

Table II
Latin Square Matrices for Four Variables

Variable No.	Matrix				
	0	1	2	-2	-1
	1	2	-2	-1	0
	2	-2	-1	0	1
	-2	-1	0	1	2
1	-1	0	1	2	-2
	2	-2	-1	0	1
	-1	0	1	2	-2
	1	2	-2	-1	0
	-2	-1	0	1	2
	0	1	2	-2	-1
	2	-2	-1	0	1
	-1	0	1	2	-2
	1	2	-2	-1	0
2	-2	-1	0	1	2
	1	2	-2	-1	0
	2	-2	-1	0	1
	-2	-1	0	1	2
	-1	0	1	2	-2
	0	1	2	-2	-1
	-2	-1	0	1	2
	1	2	-2	-1	0
	-1	0	1	2	-2
3	2	-2	-1	0	1
	-1	0	1	2	-2
	-2	-1	0	1	2
	2	-2	-1	0	1
	1	2	-2	-1	0
	0	1	2	-2	-1
	-1	0	1	2	-2
	-2	-1	0	1	2
	2	-2	-1	0	1
4	1	2	-2	-1	0
	-2	-1	0	1	2
	1	2	-2	-1	0
	-1	0	1	2	-2
	2	-2	-1	0	1

The data space is obtained by finding the mid-point and an appropriate step size for each variable. The first element of the data space is obtained by multiplying each variable's step size by element (1,1) of that variable's matrix and adding to the mid-point. The second design point is obtained by multiplying each variable's step size by element (2,1) and adding to the mid-point. This procedure is continued until the space is complete.

Several problems are possible when using the simple latin square technique. The most significant problem is that the selected set of data points may not be independent as shown in Fig. 1 (Ref 6:59). The bad fit does not adequately represent the space, while the good fit does. The lack of independence allows the possibility of significant cross-correlations building up between the variables; however, these cross-correlations would be detected during the surface fit procedure. These problems did not appear in this study.

Inputs and outputs to the latin square computer program are described in Appendix A.
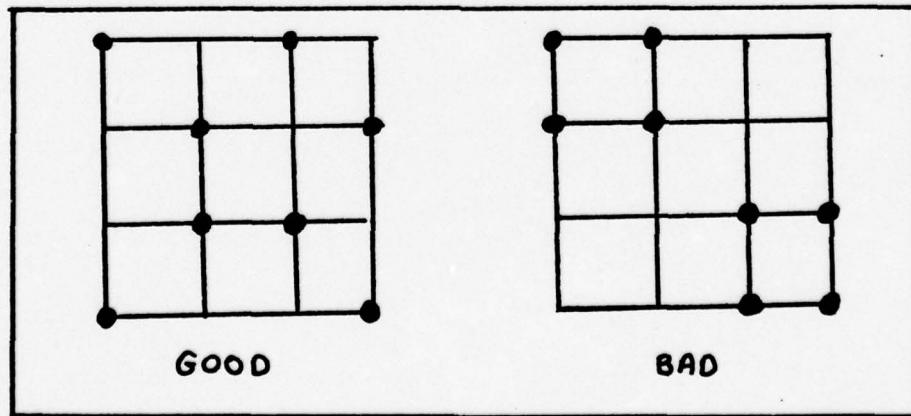

Fig. 1.   Two Examples of Latin Square Design Space
(2-Dimensional Cross-Section Views)

9

III. Mission Simulation

The mission profile used in this study is given in Fig. 2 from Ref. 5. This mission along with the independent design variables was used as input to a mission simulation program used by the AFFDL Design Branch. The CISE program was used to calculate the TOGW required to accomplish the specified mission given the set of design variables.

The mission was based on a typical interdiction mission which an advanced tactical fighter might be required to fulfill. The vehicle was to perform the mission with a one man crew and was to carry 5500 lbs of stores. The only requirement on the combat performance was the fuel allowance of fifteen percent of the total initial fuel weight (Ref 5). This requirement was met by changing the number of turns in the combat segment of the mission.

The CISE program was developed to be used as a design tool at the "zero level" of design, i.e., even before a drawing of the configuration is prepared (Ref 4:2). The program performs a weight oriented sizing process to produce the basic physical characteristics of the vehicle. The procedure was set up to use DO loops for the four design variables. An iterative process is accomplished for each combination of variables. The iteration assumes an initial TOGW and then calculates a geometry based on the guessed weight and input variables. The input mission is then "flown" and fuel needs are calculated to meet the requirements of takeoff reserves, combat penalties, and landing reserves. The final process in the iteration is to calculate the major weight components that include the wing, the body, the tail, landing gear, systems and

10
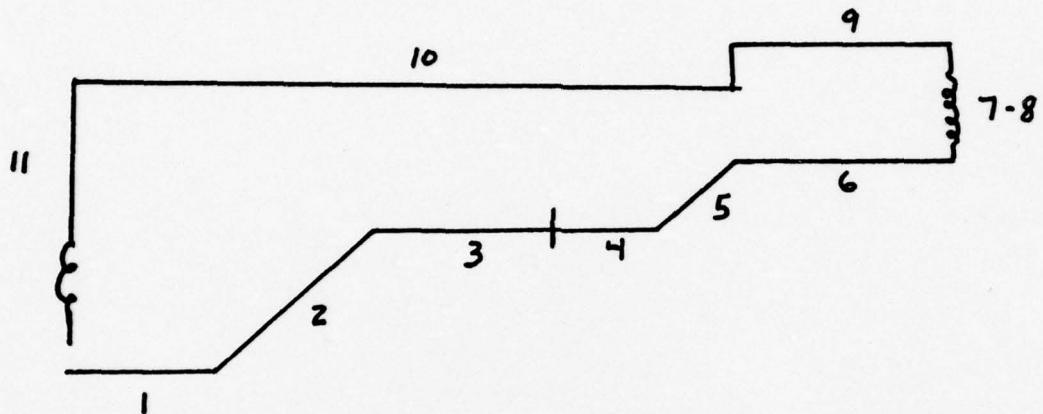
1. Warm up and takeoff

2. Climb to 45000 ft

3. Cruise at .8M for 200 NM

4. Accelerate to 1.6M

5. Climb to 50000 ft

6. Cruise at 1.6M for variable distances (150, 200, 250, 300, 350 NM)

7. Combat (15% of initial fuel for combat penalty)

8. Drop stores

9. Cruise at 1.6M at 55000 ft for same distance as Item 6

10. Decelerate to .8M and cruise back at 45000 ft

11. Loiter at sea level for 20 minutes and land

Fig. 2. Mission Profile

11

equipment, and payload. These group weights are summed to yield a TOGW at the end of the iteration cycle. When this weight is within a specified tolerance of the initial guess, then the process is terminated and another design point is initiated. If the weight is not within the tolerance, then it is used as the initial guess and the process is repeated.

The procedure that would normally be followed in the design process using surface fit approximations would be to input the latin square design space of independent variables (in this particular study the 45 design points of Table III) to the mission simulator which would output the dependent variables (in this particular study the TOGW's and DISTL's of Table IV). These steps would be followed by a surface fit procedure and an optimization routine (both to be discussed in later chapters). However, this normal procedure was deviated from slightly in this study. It was desired to have a basis for judging the "goodness and accuracy" of the surface fit approximations. In order to accomplish this goal, the entire design space consisting of all 625 design points were input to the CISE program which output 625 TOGW's and DISTL's. Those points which had been selected by the latin square technique (Table III) were included in the 625 design points; hence, the 45 output functions (TOGW's and DISTL's) for the latin square design space were already available from having run the simulation for the complete design space.

There would have been a slight difference in the values of the output functions for the 45 latin square design points had they been the only data points input to CISE as is the case in the normal design procedure described above. This is due to the weight tolerances and initial guess in the iteration process. These differences were negligible.

12

The TOGW's for the selected design space are given in Table III in the results section.

Input and output requirements for the CISE program are given in Appendix B.

IV. Surface Fit Procedure

The next step in the process was to perform a regression analysis on the performance function data. A second order polynomial was to be generated relating the TOGW to the four design variables, i.e.,

$$TOGW = A_o + \sum_{i=1}^{n} A_i X_i + \sum_{j=1}^{n} \sum_{i=1}^{n} A_{ji} X_i X_j \qquad (1)$$

where the A's are the coefficients and the X's are the four design variables. The design variables were input in a normalized form so that all values would lie on the interval (-1,1). This was done by using the following transformation:

$$X' = \frac{X - \frac{1}{2} (X_{MAX} + X_{MIN})}{\frac{1}{2} (X_{MAX} - X_{MIN})} \qquad (2)$$

This normalization was necessary to eliminate round-off errors which would have occurred due to the difference in magnitude of the variables. The normalized variables were also used in the optimization routine and transformation to real range values occurred when an optimal design point had been found.

The quadratic approximation was chosen for two basic reasons. The first reason was that the quadratic polynomial is simple and easy to work with and makes possible the easy and economical calculation of partial derivatives used in the optimizer. Adequate representation can be obtained with second order surfaces thus avoiding higher order approximations (Ref 7:595). The second reason is more mathematically

14

rational. Basically, the second order assumption is equivalent to assuming that the performance function can be adequately approximated by the first few terms of its Taylor series expansion about some nominal design point (x_1, x_2, x_3, x_4). The basic requirement is that the range of values for the design variables be kept reasonably small. If this is the case, then the quadratic polynomial derived from the regression analysis will provide adequate approximations within these ranges (Ref 2:4).

The number of unknown regression coefficients (K) in the quadratic polynomial of Eq (1) is $(n+1)*(n+2)/2$ where "n" is the number of independent variables. The number of design data points must be greater than or equal to the number of unknown coefficients so that an over-determined system of linear equations in the form of Eq (1) in terms of the unknown coefficients A_0, A_1, and A_{j1} can be solved by the method of least squares. The least squares method selects the terms so as to minimize the error (or residual) sum-of-square, SSE,

$$ SSE = \sum_{i=1}^{N} \epsilon_K^2 \tag{3} $$

where ϵ_K is the difference between the true value of the performance function and the approximated value and where N is the number of data points input to the regression analysis.

The goodness-of-fit of the regression surface is tested statistically by variance analysis. The regression program used in this study employed four tests to determine the goodness-of-fit. These were the standard F-statistic for regression, the multiple correlation coefficient squared, the significance ratio, and the standard error.

15

The F-statistic is defined as the ratio of the regression mean square,

$$MSR = \frac{\sum\limits_{k=1}^{N} (TOGW_k - \overline{TOGW})^2}{N} \qquad (4)$$

to the error mean square,

$$MSE = \frac{\sum\limits_{k=1}^{N} (TOGW_k - TOGW_k)^2}{(N - K)} \qquad (5)$$

where $\overline{TOGW}$ is the mean of the true TOGW's in the N data points, $TOGW_k$ is the TOGW predicted by the polynomial at data point k, and K is the number of coefficients in Eq (1). The F value calculated should exceed the F value found in the standard tables for K and N-K degrees of freedom at the 95% confidence level to ensure a good fit (Ref 6:595). If this test is met, then a significant regression is assumed.

The multiple correlation coefficient squared (MCC^2 or R^2) is defined as

$$MCC^2 = \frac{\sum\limits_{k=1}^{N} (TOGW_k - \overline{TOGW})^2}{\sum\limits_{k=1}^{N} (TOGW_k - \overline{TOGW})^2} \qquad (6)$$

where the terms are as defined for Eqs (4) and (5). This quantity varies between 0 and 1 and a value close to 1 indicates that the regression equation tracks the data quite well. It is desirable to have a value close to 1.

The standard error is

$$STERR = \sqrt{MSE} \tag{7}$$

and is a measure of goodness-of-fit. The smaller the value of the standard error the better the approximation.

Information on the significance ratio used in the regression program was limited to the fact that it should be as large as possible in order to ensure a good fit. The significance ratio was used to determine which step in the regression analysis would be used to select the coefficients for the quadratic approximation. A forward and backward regression analysis was performed, i.e., variables were entered and removed whenever a better fit could be obtained. The second order and cross product terms were considered as separate variables in this part of the problem; that is, (WOS^2) could be entered at a step even though WOS was not in the equation. The step chosen to be the most representative was based on maximizing

$$FACT = \sqrt{\frac{SIG\ RATIO}{STERR}} \times MCC^2 \tag{8}$$

Input and output requirements for the regressor are found in Appendix C.

17

V. Surface Fit Optimization

The basic optimization problem considered was to minimize or
maximize some performance function subject to constraints on other
performance functions. A maximization problem can be converted to a
minimization problem simply by taking a negative of the function to be
maximized and then performing a minimization (i.e., RANGE is maximized
by minimizing -RANGE). Similarly, inequality constraints can all be
regarded as less than or equal to constraints. This is accomplished
by taking the negative of a function that is to be greater than some
value and making it less than the negative of this value (i.e.,
RANGE $\geq$ 150 would be rewritten -RANGE $\leq$ -150 where -RANGE would be
defined as a new function). Thus, using these conventions, the
optimum design problem can be rewritten as

$$\text{Minimize} \quad f_1 \, (\underline{x})$$
$$\text{Subject to} \quad f_1 \, (\underline{x}) \leq c_i \; (i=2,\ldots,m) \tag{9}$$

where $\underline{x}$ is the vector of independent variables, f's are the perform-
ance functions, and the c's are the fixed numerical values defining
the upper limits on the constraint functions. The independent
variables are also constrained to lie within a region of interest,
within which an optimal design is supposed to lie. This region is the
area determined in the selection of the design space and is the region
in which the quadratic approximation is valid.

The penalty function method has been shown to work on the minimi-
zation problem of Eq (9) (Ref 6:29-32). This method transforms the

18

inequalities out of the problem by establishing a penalized cost
function of the form

$$F(\underline{x}) = f_1(x) + P_K \sum_{j=2}^{m} (CV_j)^2 \tag{10}$$

where CV represents the violation of the constraint inequalities; if
the constraint is satisfied then CV = 0. P_K is a weighting factor in
Eq (10) which modulates the severity of violating the constraints.
This weighting factor can be considered the tolerance level for con-
straint violations; hence, if the acceptable deviation from the con-
straints was one percent then P_K would be 100. It was assumed in this
study that all constraints required this tolerance and P_K was taken to
be a constant equal to 100.

The penalty function method has the advantage that the non-linear
constraints are removed from the optimization problem by a simple
transformation approach. It also has an advantage in that it is not
necessary to locate a feasible starting point for the optimization
search. The penalty function algorithm can always be started from a
nonfeasible point (Ref 6:31).

There are, however, two major problems in the penalty function
method. The first problem is that for large values of the weighting
factor P_K in Eq (10), the penalty cost function can develop a long
narrow valley containing the minimum point (Ref 1:40-41). Gradient
procedures for finding the minimum tend to go back and forth between
the sides of the valley instead of going down the long direction of
the valley. Another problem is that the generation of the penalty

19

function may also create new pseudo minima that are not present in the original problem. Ref 2 (p. 31) states "These difficulties are numerical in nature, corresponding to large values of P_K and/or a sharply decreasing cost function. These difficulties should not occur with the approximating functions which are used."

The form of the quadratic functions supplied by the regression analysis was

$$f_i(\underline{x}) = A_{i_0} + A_{i_1}^T(\underline{x}) + \tfrac{1}{2}\underline{x}^T A_{i_2} \underline{x} \quad (i=1,\ldots,m) \quad (11)$$

where A_{i_0} is a constant term, A_{i_1} is an "n" vector, and A_{i_2} is an n x n symmetric matrix. The form of this equation suggests the use of a gradient type search procedure since the gradient of the cost function would be readily available. Indeed, the desire to use a gradient method was one of the reasons for attempting the surface fit. The gradient of the penalty cost function is

$$\underline{g}(\underline{x}) = A_{11} + A_{12}\underline{x} + 2P_K \sum_{j=2}^{m} (CV_j)(A_{j_1} + A_{j_2}\underline{x}) \quad (12)$$

A conjugate gradient technique was used in this study. The ordinary gradient has the drawback of slow convergence and methods have been developed to reduce the number of iterations required for convergence (Ref 9:10). These methods use a modified search direction to speed up the convergence rate.

The minimizing algorithm used was as follows: (a) an evaluation of the gradient ($\underline{g}_i$) at a given point $\underline{x}_i$ was accomplished, (b) the search direction $\underline{p}_i$ was computed according to

20

$$\underline{p}_i = H_i^T \underline{g}_i \tag{13}$$

where "i" denotes the present point and H_i was given by

$$H_i = H_{i-1} + \frac{(\Delta \underline{x}_{i-1} - H_{i-1} \Delta \underline{g}_{i-1})(\Delta \underline{x}_{i-1} - H_{i-1}^T \Delta \underline{g}_{i-1})^T}{(\Delta \underline{x}_{i-1} - H_{i-1}^T \Delta \underline{g}_{i-1}) \Delta \underline{g}_{i-1}} \tag{14}$$

where i-1 denotes the previous point, H_0 was defined to be the unity matrix, $\Delta \underline{g}_{i-1} = \underline{g}_i - \underline{g}_{i-1}$, and $\Delta \underline{x}_{i-1} = \underline{x}_i - \underline{x}_{i-1}$, (c) a one dimensional search was performed on the function $F(\underline{x}_{i+1}) = F(\underline{x}_i + \Delta \underline{x}_i)$ where

$$\Delta \underline{x}_i = -t_i \underline{p}_i \tag{15}$$

and the search was to minimize $F(\underline{x}_i - t_i \underline{p}_i)$ with respect to t_i (this one dimensional search will be discussed later), (d) having found the t_i that yielded a minimum, the displacement $\underline{x}_i$ was computed according to Eq (15), (e) the next point, $\underline{x}_{i+1}$ was obtained by

$$\underline{x}_{i+1} = \underline{x}_i + \Delta \underline{x}_i \tag{16}$$

The value of $F(\underline{x}_{i+1})$ will always be less than the value of $F(\underline{x}_i)$ (Ref 8:271). These steps (a-e) were repeated until the stopping condition

$$\underline{g}_i^T \underline{g}_i \leq 10^{-6} \tag{17}$$

was met. This stopping condition may correspond to either a minimal point or a nonminimal stationary point (Ref 8:275). For this reason, small perturbations about this point were investigated.

21

The entire algorithm may need to be restarted by setting $H_1 = H_0$. This is required when the inequality

$$\left| g_1^T \, p_1 \right| \;\leq\; 10^{-6} \tag{18}$$

is satisfied. If the algorithm is not restarted on this condition then the starting point for the one-dimensional search will automatically satisfy the search stopping condition of Eq (22) so that $t_1 = 0$.

The linear search method was that proposed by Davidon and has been used by Fletcher and Powell (Ref 3). The method was (a) to estimate the order of magnitude of the stepsize t_1, (b) establish bounds on t between which the minimum point lies, and (c) to perform a cubic interpolation to find t_1.

The determination of the order of magnitude of t_1 was based on an available estimate (est) of the value of $F(\underline{x})$ at the unconstrained minimum. This estimate was available in the problem at hand and would be expected to be around -1 when normalized variables were being used. The step size was determined by

$$h = k = \frac{2(F(\underline{x}_1) - est)}{\underline{p}_1^T \, \underline{g}_1} \quad \text{if } 0 < k < \frac{1}{\sqrt{\underline{p}_1^2}} \tag{19}$$

or

$$h = \frac{1}{\sqrt{\underline{p}_1^2}} \quad \text{otherwise.}$$

The bounds on t_1 were established by examining

$$y(t) = F(\underline{x}_1 - t \, \underline{p}_1)$$
$$y'(t) = -\underline{p}_1^T \, \underline{g} \, (\underline{x}_1 - t \, \underline{p}_1) \tag{20}$$

22

at points where $t = 0$, h, 2h, 4h, ..., a, b where b is the first point
at which either y' is non-negative or y has not decreased. It then
follows that t_i is bounded in the interval (a,b).

Given these limits on t_i a cubic interpolation estimating t_i may
be given by

$$t_e = b - (\frac{y'(b) + w - z}{y'(b) - y'(a) + 2w}) (b - a)$$

$$z = 3 (\frac{y(a) - y(b)}{b - a}) + y'(a) + y'(b) \tag{21}$$

$$w = \sqrt{z^2 - y'(a) \, y'(b)}$$

If neither y(a) nor y(b) is less than $y(t_e)$ then t_e is accepted as the
estimate of t_i and the stopping condition is tested at t_e. If the
condition is met, the one dimensional search is terminated. If the
condition is not met, the cubic interpolation is repeated over the
sub-interval (a, t_e) if $y'(t_e)$ is positive and the cubic interpolation
is repeated over the sub-interval (t_e, b) if $y'(t_e)$ is negative. The
stopping condition is determined by precision requirements and was

$$\left| \underline{g}_{i+1}^T \, \underline{p}_i \right| \le 10^{-6} \tag{22}$$

where the constraint limit must be the same as in Eq (18).

Two different minimization problems were investigated. The first
was to minimize the TOGW subject only to the constraints on the inde-
pendent design variables. In solving this problem, the design
variables were entered into the penalty cost function by creating

23

false performance functions consisting only of the variables and their limits. The second problem was to minimize the TOGW subject to a constraint on the landing distance which was a performance parameter available from CISE.

Input and output requirements for the optimization routine are described in Appendix D.

VI. Results

The design space selected by the Latin Square technique is given in Table III. Each design point was input to the mission simulator and the performance values (TOGW and DISTL) for each point are given in Table IV. These 45 points were then input to the regression analysis. The second order polynomial generated for each performance function was (equations are in terms of normalized values)

$$\text{TOGW} = -.31638 + .06307 \text{ (WOS)} - .04207 \text{ (AR)} + .79413 \text{ (TW)}$$
$$+ .18533 \text{ (RN)} + .03678 \text{ (WOS}^2) - .04107 \text{ (WOS)(AR)}$$
$$- .04316 \text{ (WOS)(TW)} - .03023 \text{ (AR)(TW)} - .31439 \text{ (TW}^2)$$
$$+ .07082 \text{ (TW)(RN)}$$

$$(23)$$

$$\text{DISTL} = .01156 + .87538 \text{ (WOS)} - .10542 \text{ (AR)} - .05551 \text{ (TW)}$$
$$- .08192 \text{ (RN)} - .02251 \text{ (WOS}^2) - .01018 \text{ (WOS)(AR)}$$
$$- .01253 \text{ (WOS)(TW)} - .02614 \text{ (WOS)(RN)} - .00507 \text{ (AR}^2)$$
$$+ .00966 \text{ (AR)(TW)} + .00488 \text{ (AR)(RN)} - .02229 \text{ (TW}^2)$$
$$+ .00287 \text{ (TW)(RN)}$$

The statistical analysis for each equation is given in Table V. The value of the F-statistic from the standard tables for a numerator of 45 degrees of freedom and a denominator of 15 degrees of freedom is 2.19. The TOGW equation does meet this requirement so it has a 95% confidence level that it represents the true data space. The other values for the TOGW equation also indicate a good fit. The F-value for the DISTL equation was lower than the required value; however, the

Table III
Selected Latin Square Design Space

Case No.	WOS	AR	TW	RN
1	90	3.5	.9	250
2	100	4.5	.5	200
3	110	3.0	1.1	150
4	70	4.0	.7	350
5	80	2.5	1.3	300
6	110	4.0	.7	150
7	80	4.5	.5	300
8	100	2.5	1.3	200
9	70	3.0	1.1	350
10	100	4.0	1.1	300
11	110	2.5	.7	250
12	70	3.5	1.3	200
13	80	4.5	.9	150
14	90	3.0	.5	350
15	70	4.5	.9	200
16	90	2.5	.7	350
17	110	3.0	.5	250
18	80	3.5	1.3	150
19	110	4.5	1.3	350
20	70	3.0	.9	300
21	80	4.0	.5	250
22	90	2.5	1.1	200
23	100	3.5	.7	150
24	80	2.5	1.1	250
25	100	3.0	.9	150
26	70	3.5	.7	300
27	90	4.0	.5	200
28	70	2.5	.5	150
29	80	3.5	1.1	350
30	90	4.5	.7	300
31	100	3.0	1.3	250
32	110	4.0	.9	200
33	90	3.0	1.3	300
34	110	3.5	1.1	200
35	80	4.0	.9	350
36	100	4.5	.7	250
37	80	3.0	.7	200
38	90	4.0	1.3	150
39	100	2.5	.9	350
40	110	3.5	.5	300
41	70	4.5	1.1	250
42	100	3.5	.5	350
43	70	4.0	1.3	250
44	90	4.5	1.1	150
45	110	2.5	.9	300

Table IV
Mission Simulation Output

Case No.	TOGW	DISTL
1	35407	6230
2	31751	6831
3	38158	7734
4	33579	4721
5	45299	5514
6	32253	7655
7	31567	5380
8	43695	6981
9	40411	4812
10	39904	6671
11	33844	7745
12	42660	4812
13	33956	5488
14	32604	6246
15	34885	4746
16	33936	6307
17	33302	7690
18	41480	5560
19	45256	7091
20	36169	4879
21	31117	5522
22	38776	6393
23	31597	7096
24	39253	5655
25	34302	7144
26	32873	4852
27	31240	6258
28	30071	5136
29	40393	5409
30	32821	6033
31	44253	6837
32	34889	7530
33	44842	6129
34	38751	7550
35	36755	5354
36	32248	6763
37	31852	5727
38	41383	6152
39	37548	6916
40	33546	7513
41	38687	4691
42	33215	6805
43	43414	4703
44	37176	6124
45	37297	7625

other statistical values are extremely good and the maximum difference
that the equation predicts from the true data is only six percent.

Table V
Statistical Analysis of Selected Equations

	F-value	Significance Ratio	Multiple Correlation	Standard Error
TOGW	5.3352	922.4982	.99816	.04042
DISTL	.8058	25053.265	.99995	.00758

One problem was noted in the TOGW equation. The AR appeared only
as a linear term indicating that a proper minimum would not exist.
Also, the sufficiency condition for a minimum was not met in that the
matrix of second partial derivatives was not positive, semi-definite.

To analyze this problem, graphs were made of the TOGW versus the
AR for several values of WOS at constant RN and TW. These graphs are
presented in Figs. 3-8. It was concluded from these graphs that the
calculated curves do represent the actual data trends and yield values
of TOGW that are within two percent of the actual data.

Three dimensional plots were made of TOGW versus AR and WOS at
constant RN and TW. Once again the calculated plots exhibit the same
shaped (although they are smoother as would be expected) as the true
data and have a maximum difference of two percent. These surfaces are
presented in Figs. 9-14.

Two minimizations were performed. The first problem was to find
the minimum TOGW within the design space. The second problem was to

find the minimum TOGW subject to the constraint that the DISTL be less than 5000 feet and still be within the design space. These minimums were also found by manually analyzing the 625 data points that represented the complete data space. The minimum point for each range was found and the results are presented in Tables VI and VII.

Table VI indicates that the surface fit approximations yield minimum values of TOGW that are within two percent of the actual minimum values. The design parameters are identical with the exception of the AR. The graphs of Figs. 3-8 indicate that the TOGW is not significantly affected by changes in AR so the problem of different AR's is acceptable. Table VII shows similar results for the case when DISTL was constrained to be less than or equal to 5000 feet.

These results also indicate that the minimum design point is not inside the range of the design variables. In the unconstrained problem, the design point was at the lower limit of each of the variables. This does present a problem in that the minimum was assumed to lie within the design region. However, the approximation method did predict the same value as was found by looking at the complete space.

TOGW VS AR

RN - 150, TW - .5
ACTUAL DATA

LEGEND

□ - WOS - 70
○ - WOS - 80
△ - WOS - 90
+ - WOS - 100
× - WOS - 110

Fig. 3. TOGW vs. AR (RN = 150, TW = .5) - Actual Data

30
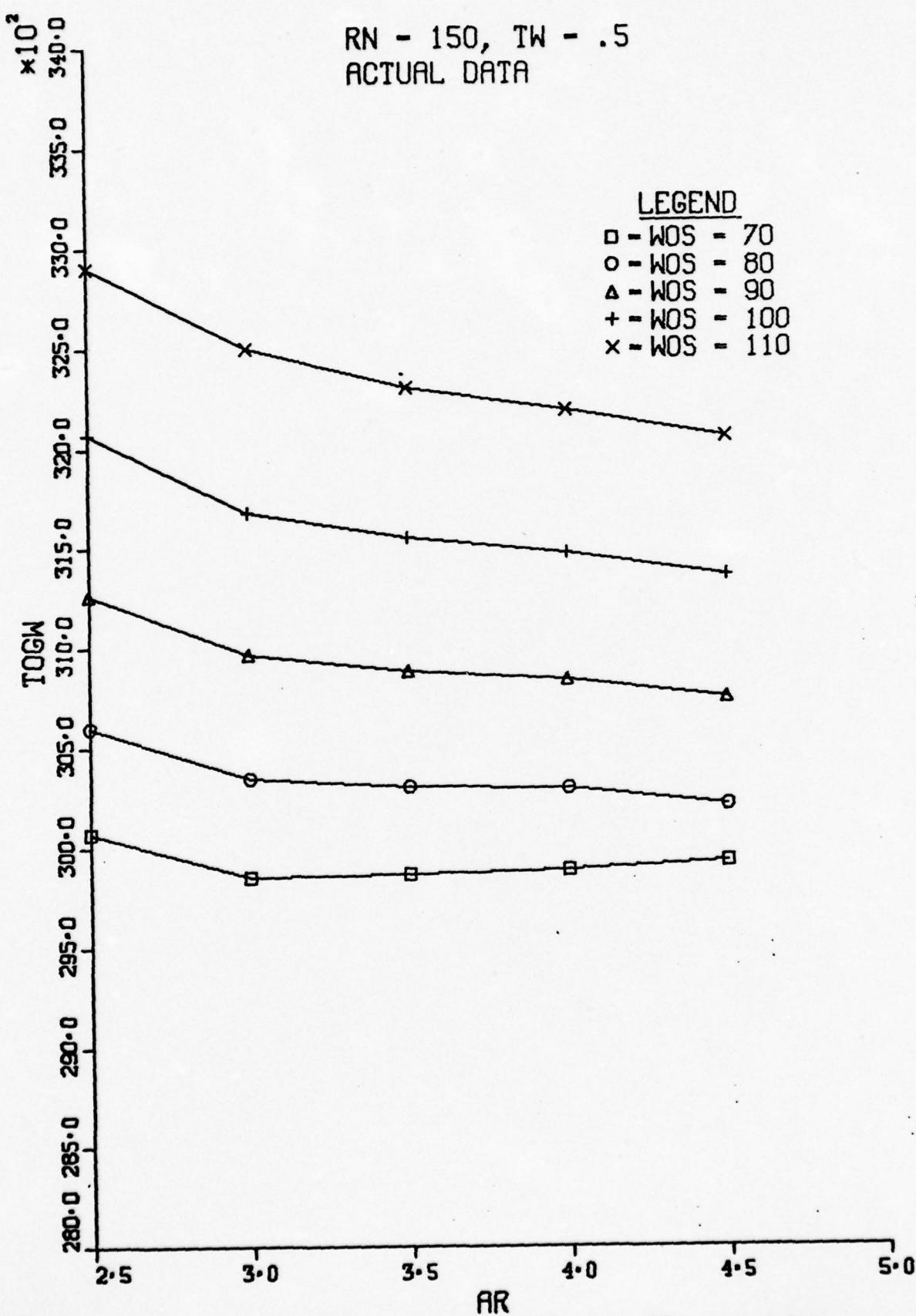
TOGW VS AR

RN - 150, TW - .5
CALCULATED VALUES
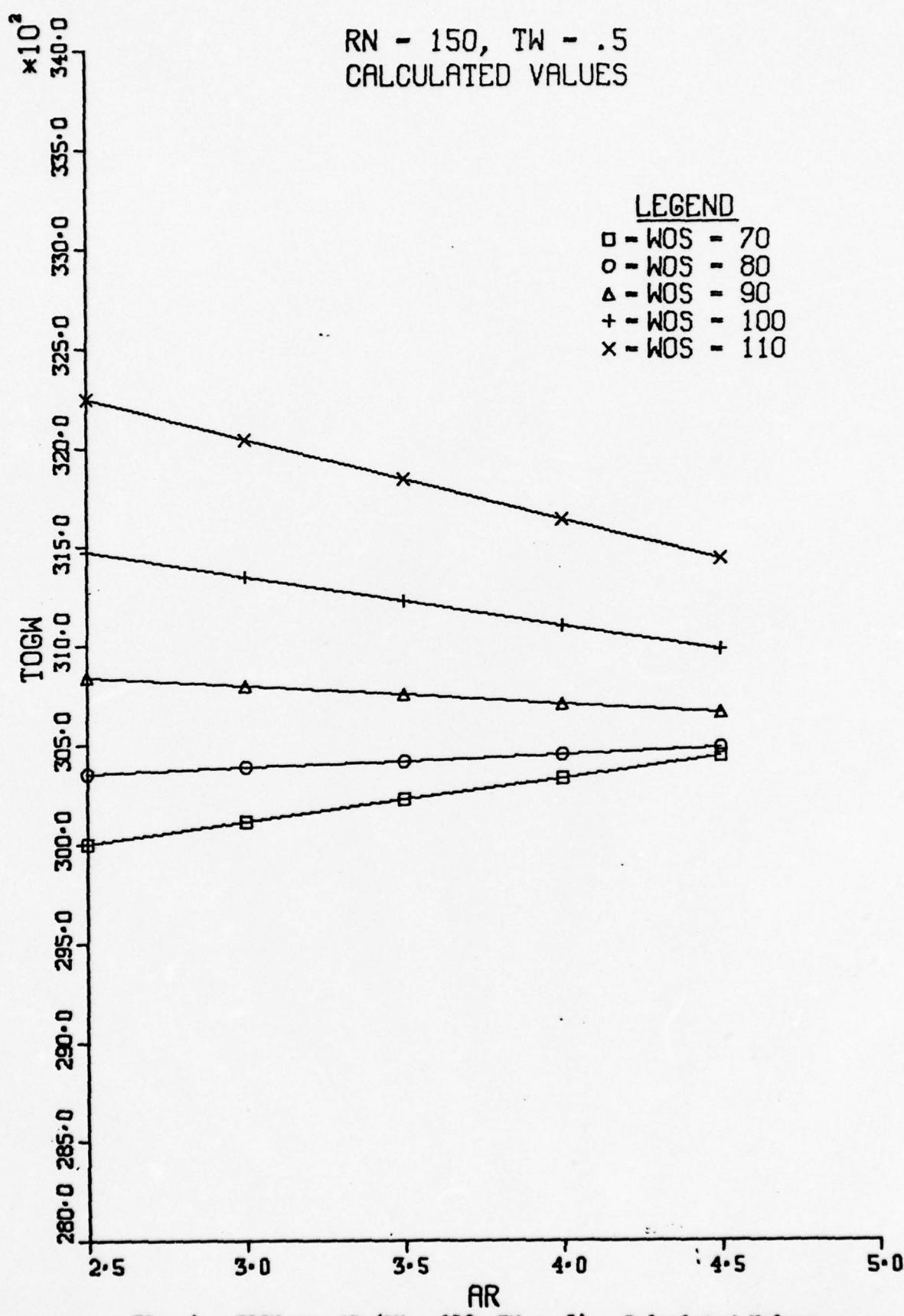
Fig. 4. TOGW vs. AR (RN = 150, TW = .5) - Calculated Values

# TOGW VS AR

RN - 250, TW - .9
ACTUAL DATA
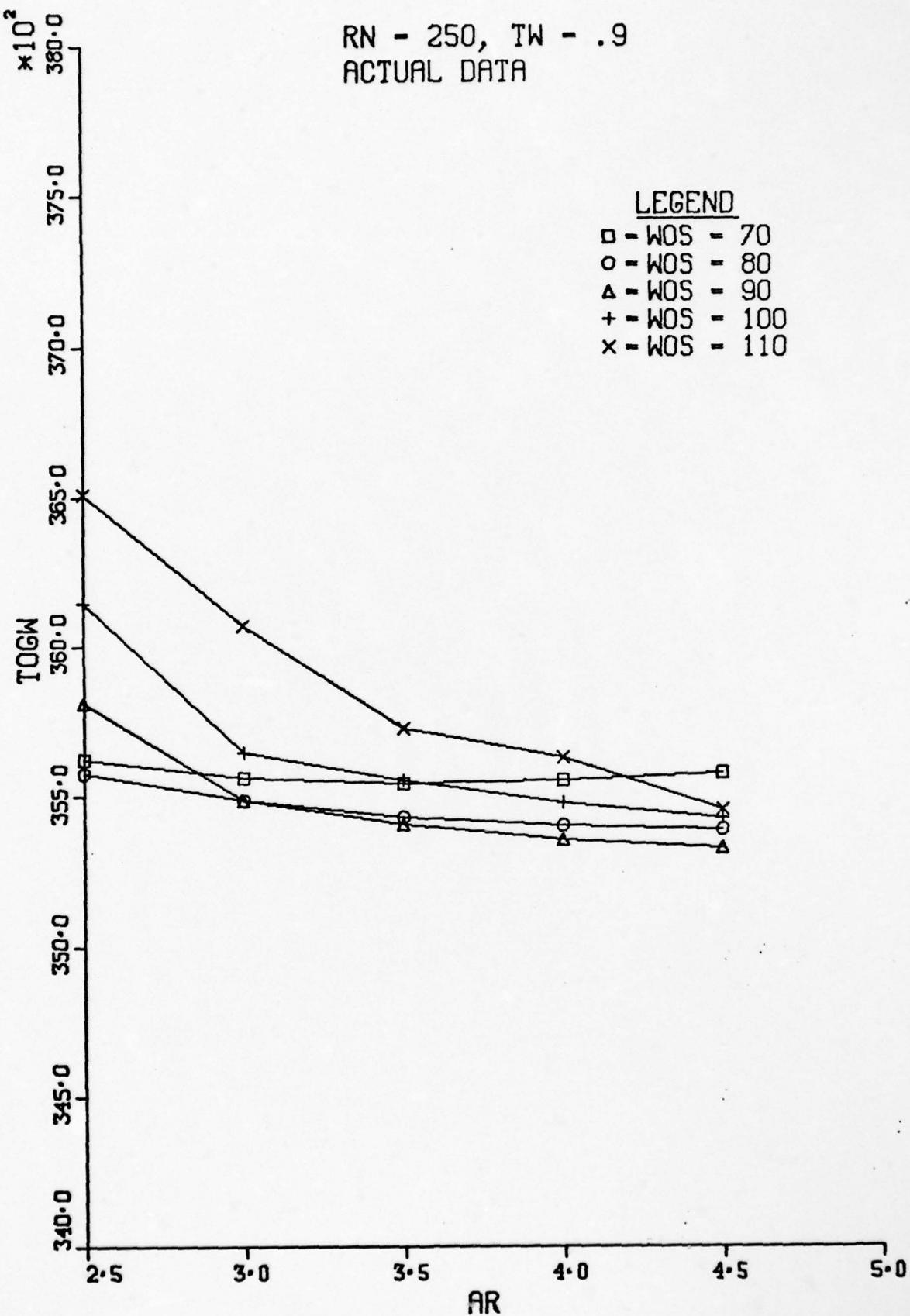
Fig. 5. TOGW vs. AR (RN = 250, TW = .9) - Actual Data

TOGW VS AR

RN - 250, TW - .9
CALCULATED VALUES
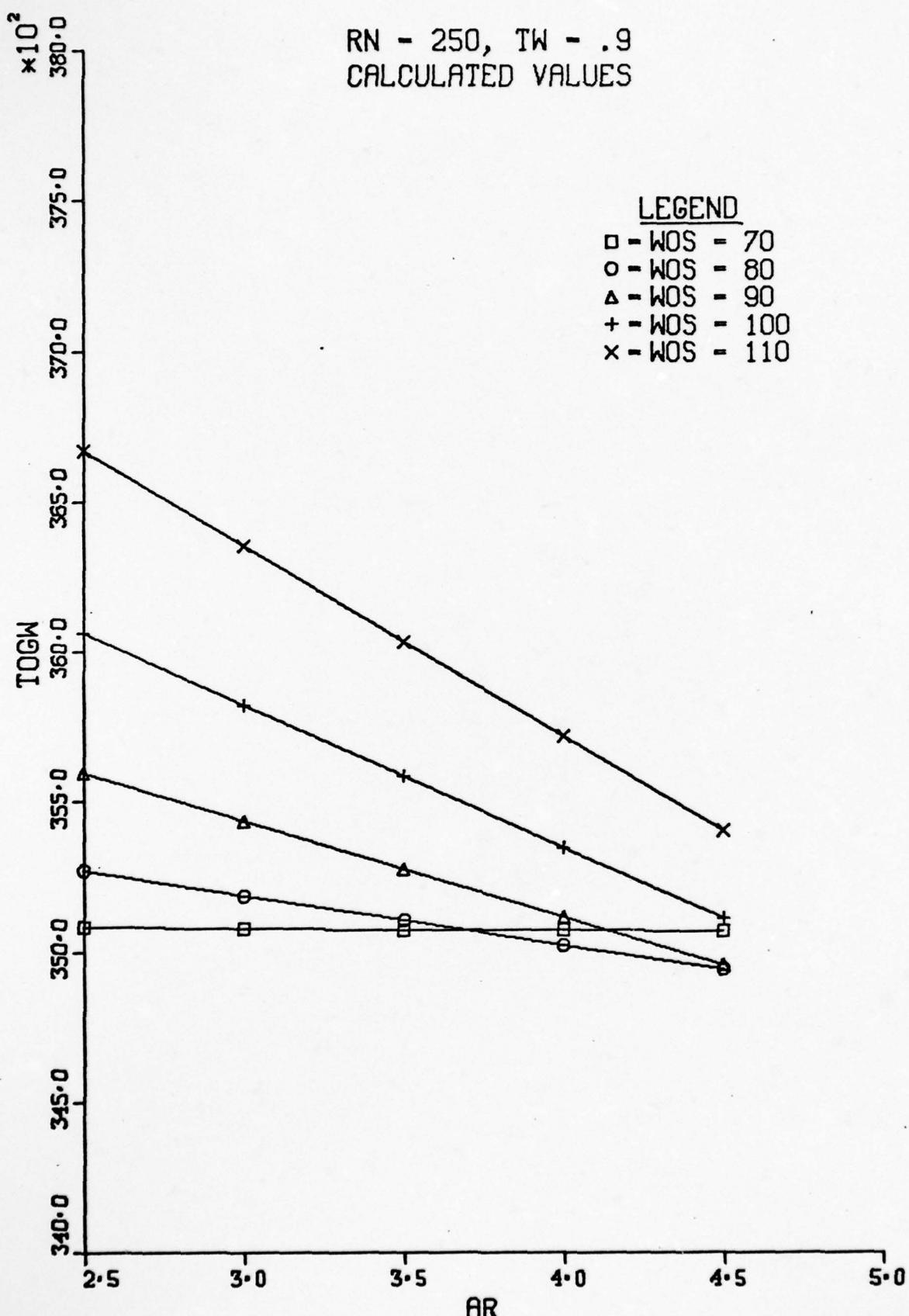
Fig. 6.   TOGW vs. AR (RN = 250, TW = .9) - Calculated Values

TOGW VS AR

RN - 350, TW - 1.3
ACTUAL DATA
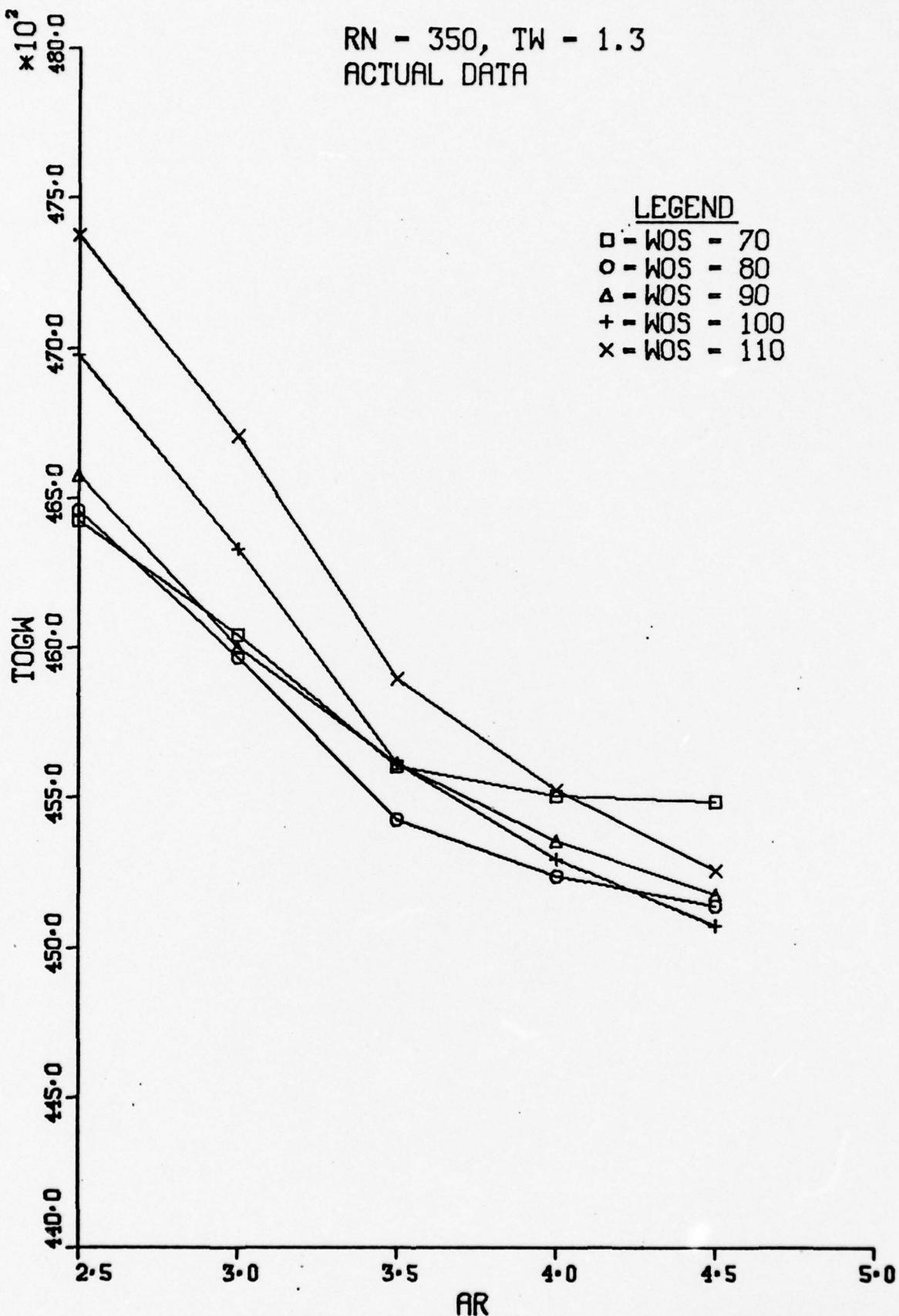
Fig. 7. TOGW vs. AR (RN = 350, TW = 1.3) - Actual Data

34

TOGW VS AR

RN - 350, TW - 1.3
CALCULATED VALUES
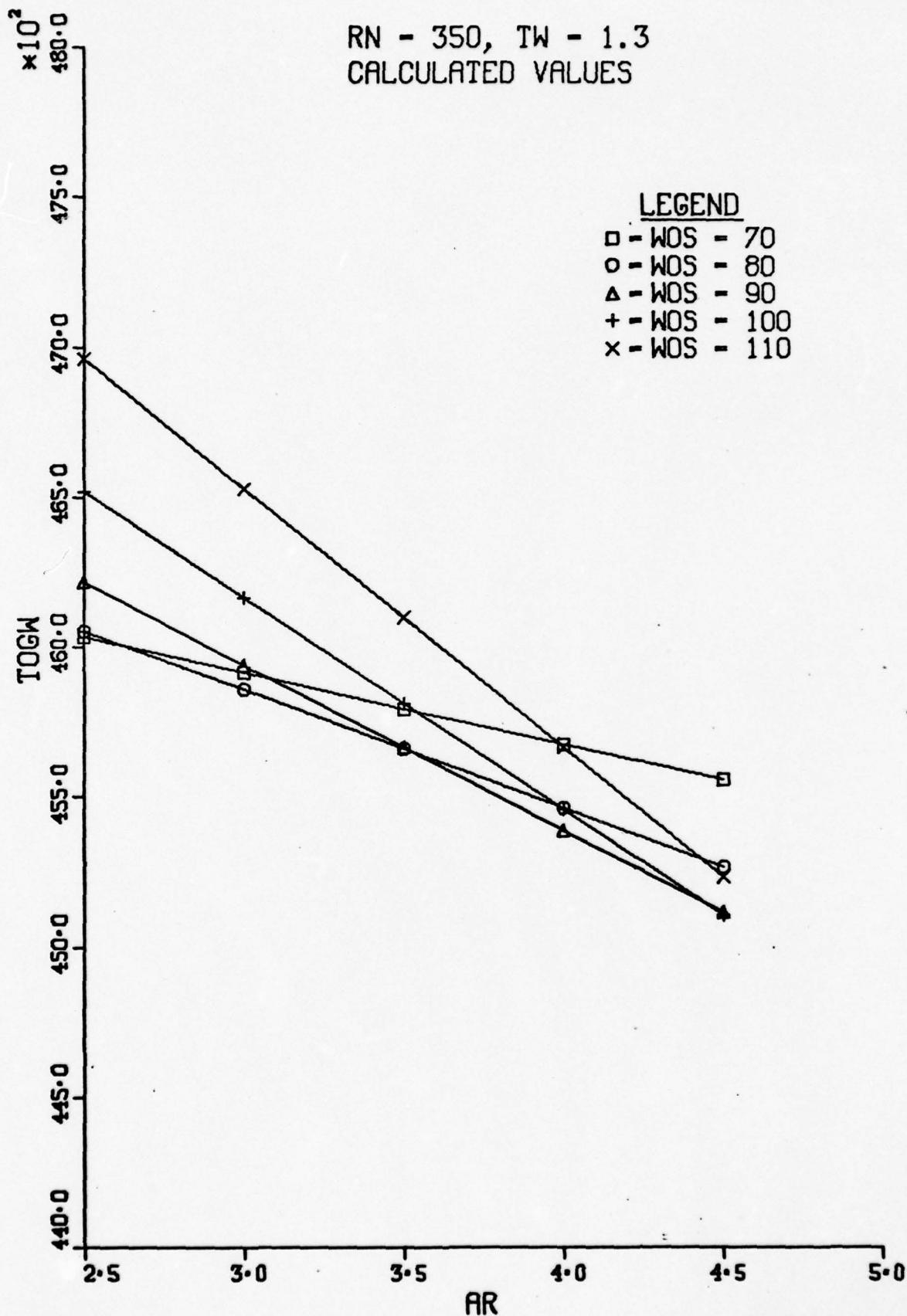
Fig. 8.   TOGW vs. AR (RN = 350, TW = 1.3) - Calculated Values

35

TOGW VS WOS — AR

RN — 150, TW — .5
ACTUAL DATA
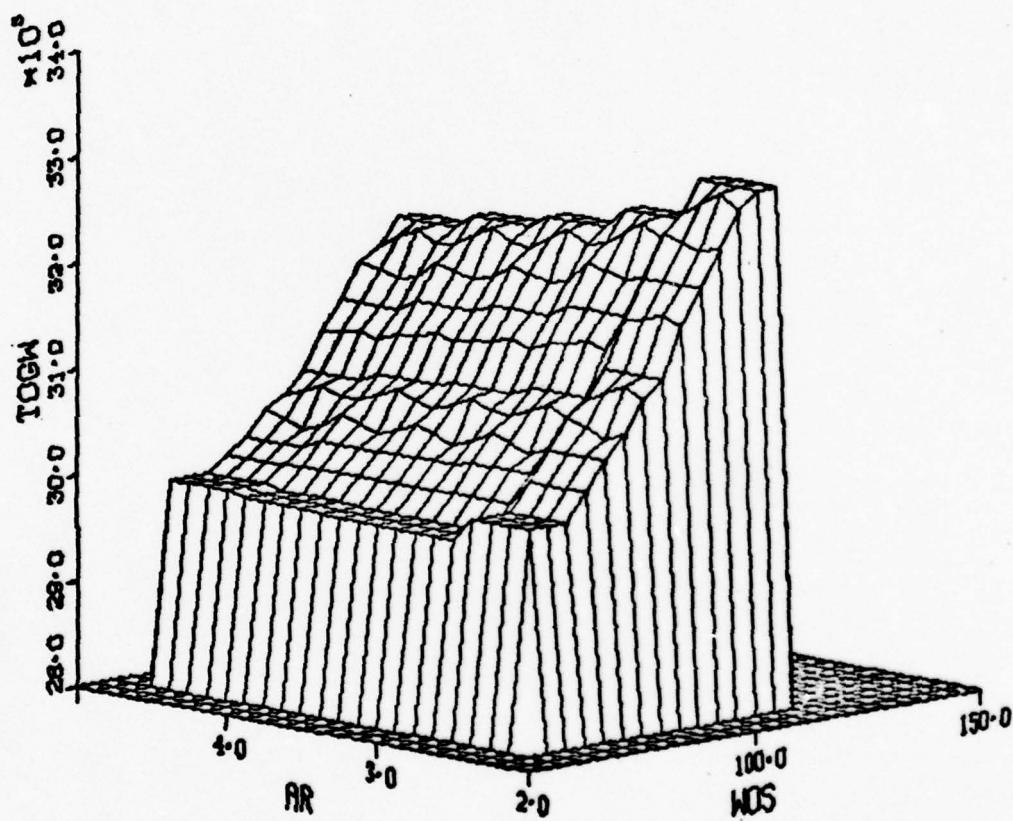
Fig. 9.   TOGW vs. WOS — AR (RN = 150, TW = .5) Actual Data

TOGW VS WOS – AR

RN – 150, TW – .5
CALCULATED VALUES
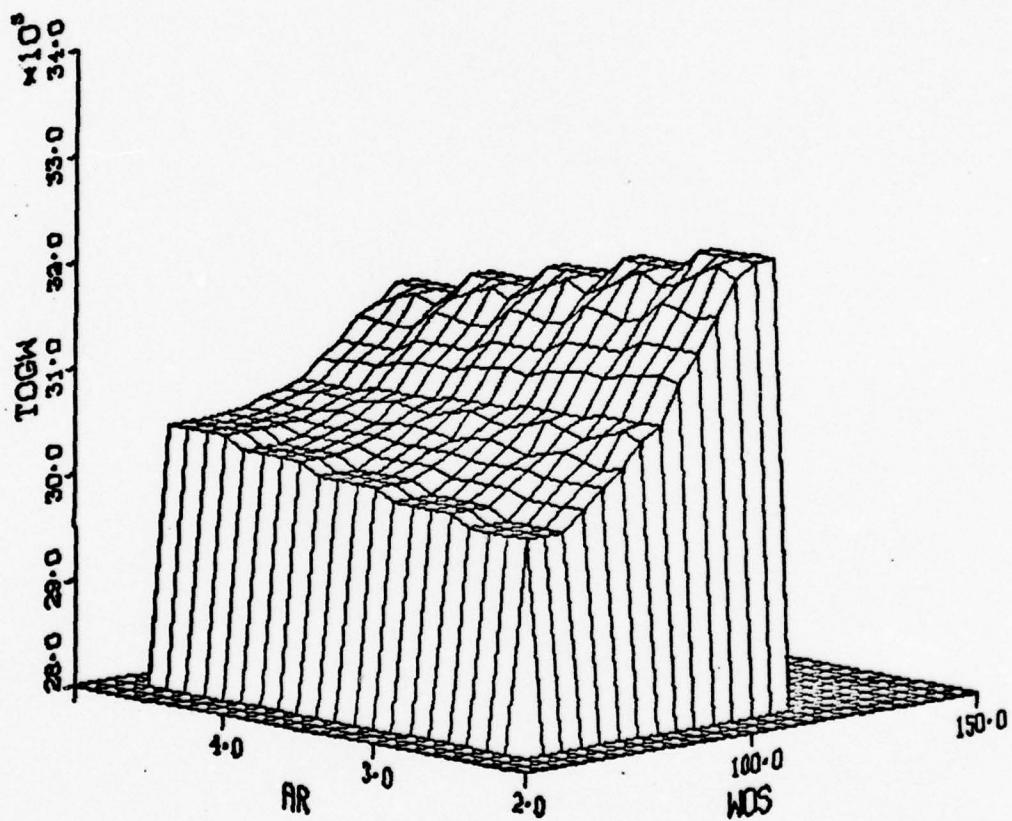
Fig. 10.   TOGW vs. WOS – AR (RN = 150, TW = .5) Calculated Values

TOGW VS WOS — AR

RN — 250, TW — .9
ACTUAL DATA
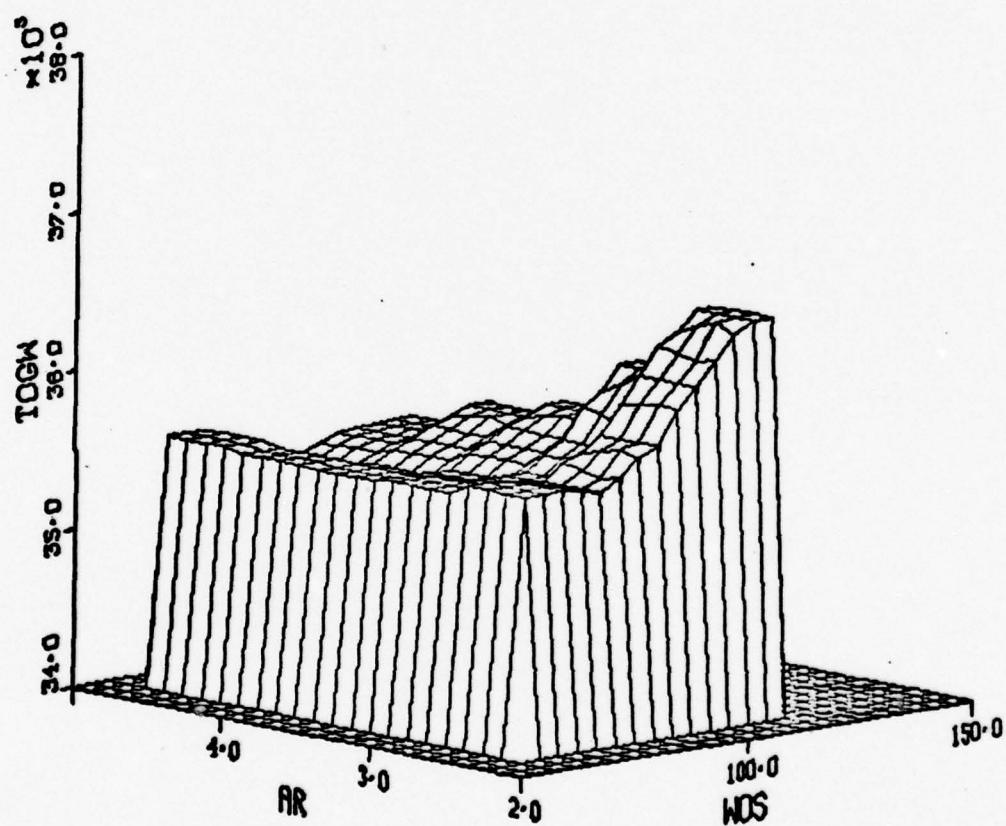
Fig. 11. TOGW vs. WOS — AR (RN = 250, TW = .9) Actual Data

TOGW VS WOS - AR

RN - 250, TW - .9
CALCULATED VALUES
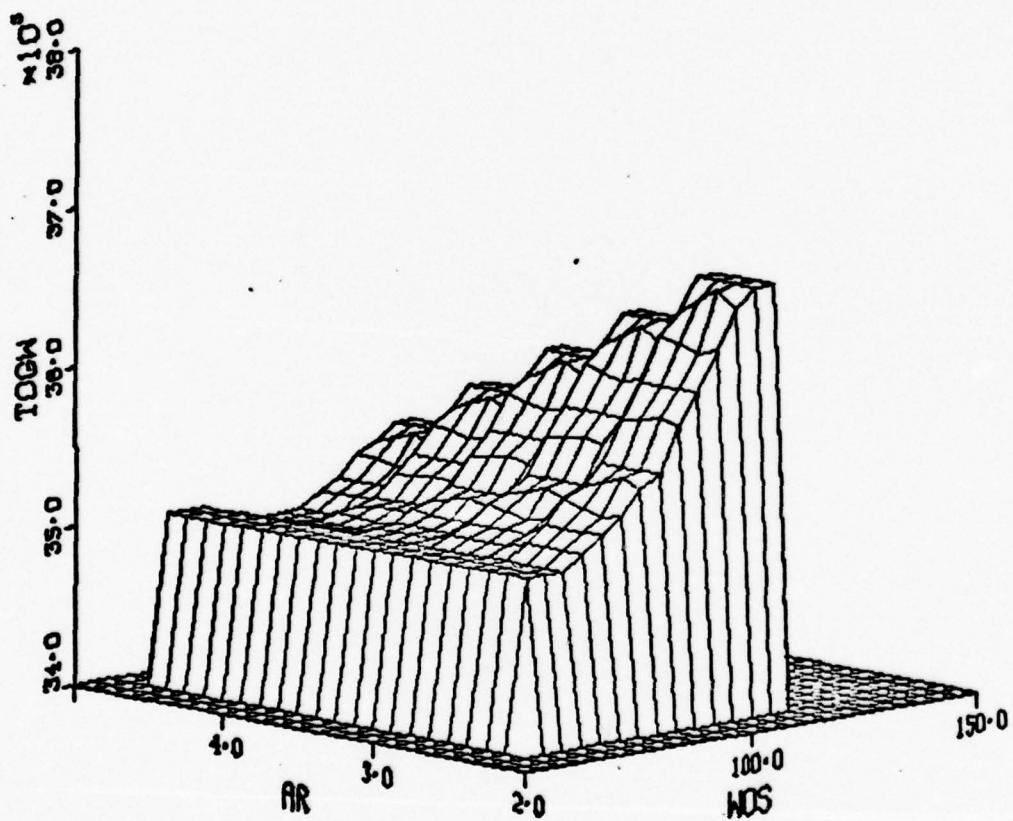
Fig. 12.   TOGW vs. WOS - AR (RN = 250, TW = .9) Calculated Values

TOGW VS WOS – AR

RN – 350, TW – 1.3
ACTUAL DATA
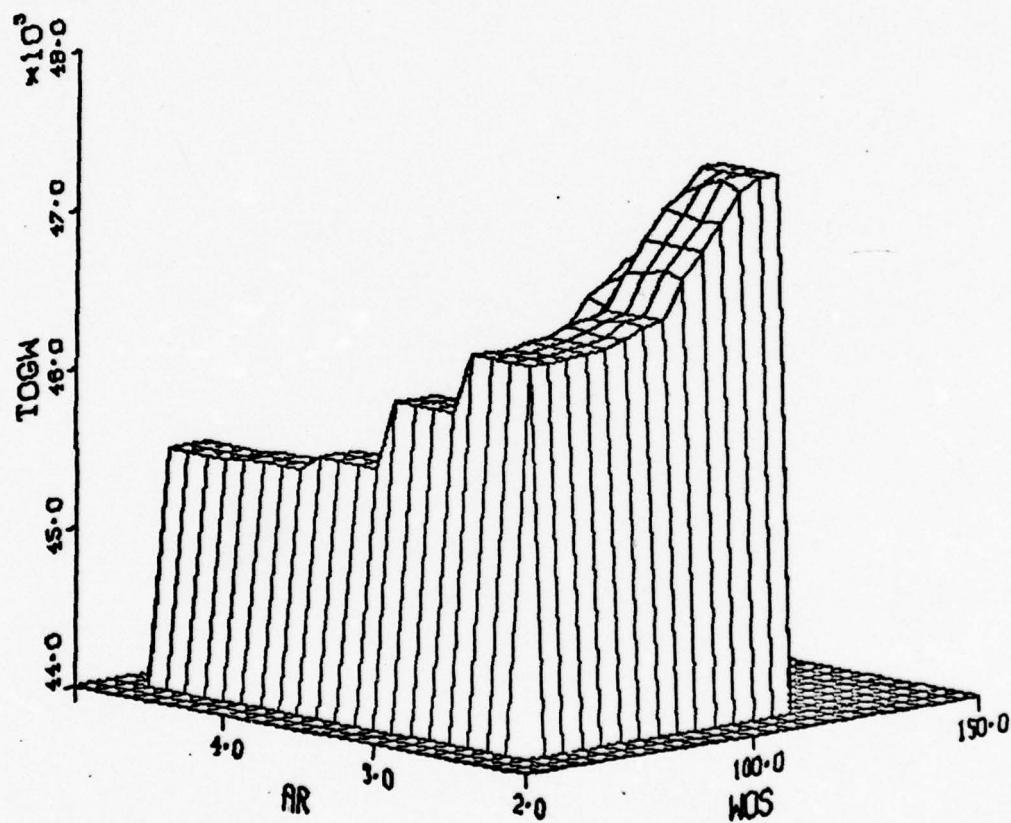

Fig. 13. TOGW vs. WOS – AR (RN – 350, TW – 1.3) Actual Data

TOGW VS WOS - AR

RN - 350, TW - 1.3
CALCULATED VALUES
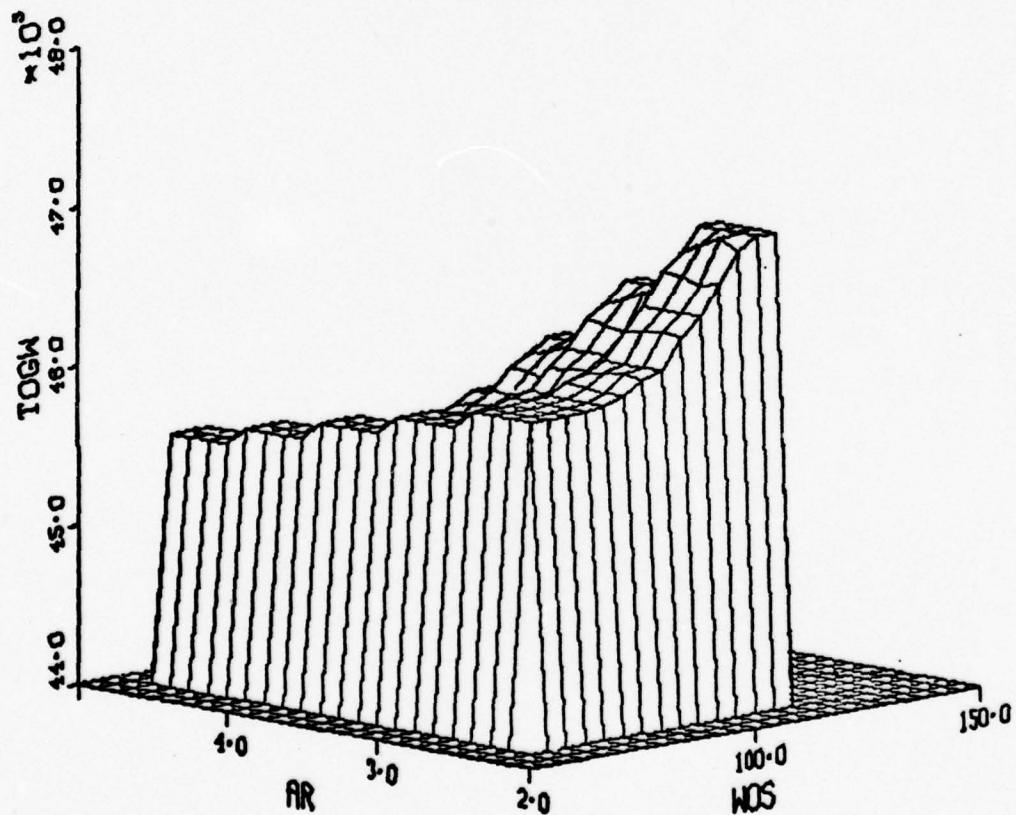
Fig. 14.   TOGW vs. WOS - AR (RN = 350, TW = 1.3) Calculated Values

Table VI
Results of the Minimization - DISTL Not Constrained

Case	RN	WOS	AR	TW	TOGW
A	150	70	3.5	.5	29864
B	150	70	2.5	.5	30000
A	200	70	3.0	.5	30295
B	200	70	2.5	.5	30436
A	250	70	3.0	.5	30774
B	250	70	2.5	.5	30872
A	300	70	3.0	.5	31282
B	300	70	2.5	.5	31308
A	350	70	3.0	.5	31818
B	350	70	2.5	.5	31743

where Case A represents the complete data space analysis; Case B represents the surface fit approximation analysis.

Table VII
Results of the Minimization - DISTL Constrained

Case	RN	WOS	AR	TW	TOGW	DISTL
A	150	70	3.5	.5	29864	5007*
B	150	70	3.5	.5	30223	4986
A	200	70	3.5	.5	30301	4947
B	200	70	3.5	.5	30659	4941
A	250	70	3.0	.5	30773	4965
B	250	70	3.0	.5	30983	4974
A	300	70	3.0	.5	31282	4905
B	300	70	2.5	.5	31308	5000
A	350	70	3.0	.5	31818	4844
B	350	70	2.5	.5	31744	4951

where Case A represents the complete data space analysis; Case B represents the surface fit approximation analysis.

*This number is within the constraint tolerance.

VII. Conclusions and Recommendations

Conclusions

1. The surface fit approximations can be used in the design process to predict designs that agree with more detailed and time consuming processes.

2. The fact that the independent variables are constrained to lie in the selected region requires that care be taken during the definition of the ranges of these variables. The ranges must be chosen small enough so that the quadratic approximations are valid; however, they must be large enough to allow the minimum point to lie within the design space. This will prevent a minimum design point which satisfies the performance function constraints from not being found.

3. The simple latin square technique as modified will yield a representative design space. This reduces significantly the number of mission simulations that must be performed; hence, computer time and engineering analysis are reduced.

Recommendations

1. Further studies should be made to demonstrate the capabilities with an expanded number of variables and performance functions. These studies should be made in close coordination with the AFFDL Design Branch using a design problem that they are considering.

2. Other design space selection techniques such as the orthogonal latin square and the D-optimal should be considered. These techniques may further reduce the number of mission simulations.

3.   It is appropriate to incorporate the surface fit approximations in the design procedures at AFFDL.   These techniques are currently being used by some aerospace companies and in particular at Boeing, enabling them to save many computer and engineering hours.

Bibliography

1. Bryson, A. E., Jr. and Y. C. Ho. <u>Applied Optimal Control</u>.
    Washington, D. C.: Hemisphere Publishing Corporation, 1975.

2. Craig, J. A., Jr. <u>An Application of D-Optimal Regression Analysis
    Design to Optimal Aircraft Design</u>. Dallas, Texas: Vought Systems
    Report, n.d. (spring 1976).

3. Fletcher, R. and C. M. Reeves. "Function Minimization by
    Conjugate Gradients." <u>Computer Journal</u>, 7:149-151 (1964).

4. Gersch, A. and D. Reichel. <u>Computerized Initial Sizing Estimate</u>.
    Fort Worth, Texas: Grumman Report No. PDR-AI-70-1 (January 1970).

5. Grumman Aircraft Company. <u>Advanced Design Composite Aircraft
    (ADCA) Final Program Review</u>. Wright-Patterson Air Force Base,
    Ohio: Contract No. F33615-75-C-3124 (August 1976).

6. Healy, M. J. <u>Surface Fit/Optimization Methodology for Aircraft
    Engine Screening</u>. Seattle, Washington: Boeing Report No.
    D180-17884-1 (January 1975).

7. Healy, M. J., <u>et al</u>. "Airplane Engine Selection by Optimization
    on Surface Fit Approximations." <u>Journal of Aircraft</u>, 12:593-599
    (July 1975).

8. Huang, H. Y. and A. V. Levy. "Numerical Experiments on
    Quadratically Convergent Algorithms for Function Minimization."
    <u>Journal of Optimization Theory and Application</u>, 6:269-282
    (No. 3, 1970).

9. Miele, A., <u>et al</u>. <u>Gradient Methods in Mathematical Programming
    Part I - Review of Previous Techniques</u>. Research supported by
    Office of Scientific Research Grant No. AF-AFOSR-828-67.
    Houston, Texas: Rice University (1969).

Appendix A

Latin Square Computer Program Input/Output

Input

Card No.	Col. No.	Description
1	12-13	Number of independent design variables (Format I2)
2-N	1-6	Alphanumeric label for the variable (Format A6)
	11-20	Lower limit for the variable (Format E10.4)
	21-30	Upper limit for the variable (Format E10.4)

"N" is the number of independent design variables plus one which allows the range of each variable to be input.

Input is assigned to Tape 5.

Output

The format of the output is (I8, 10F12.4/14X,7F12.4) which yields the data point number and the values of the design variables for that data point.

Output is assigned to Tape 6.

46

Computerized Initial Sizing Program (CISE) Input/Output

All input is in the standard Fortran Namelist format with the first character in each record appearing in the second column. The title of the design under study appears on the first data record and is in Hollerith format. The input is divided into five groups using the following names:

DESIGN, MISSION, WEIGHTS, GEOM, PROP

The items that can be input by each Namelist are:

$DESIGN

LF = Load Factor

VMAX = Maximum Equivalent Airspeed

AMMAX = Maximum Mach No.

ALTX = Altitude for AMMAX

PS = Energy Level Required at Flight Design Gross Wt.

GPS = G Level for PS

ALTPS = Altitude for PS

AMNPS = Mach No. for PS

NCREW = No. of Crew Members

NTANK = No. of 300 Gal External Tanks

NSTORX = No. of External Stores

NPYL = No. of Pylons to Carry External Stores and Tanks

NSTORI = No. of Internal Stores

CLMAX = Maximum Lift Coefficient for Takeoff and Landing

NCDX = C_{d_0} Calculation Cue; When = 0, CDOSF = C_{f_e}; When = 1,

C_{d_0} is calculated by Program and CDOSF is the Modification

Factor

CDOSF = Equivalent Skin Friction Coefficient or C_{d_0} Modification

Factor (depending on NCDX value input)

CDSTX1 = Subsonic Store Drag Modification Factor

CDSTX2 = Supersonic Store Drag Factor

ALTTOL = Altitude for Takeoff and Landing

DTEMP = Takeoff and Landing   F from Standard Day at ALTTOL

IPROP = 1 for Turbofan (P&W 401)

   = 2 for Turbojet (GE J79)

   = 3 for Reciprocating (Lycoming IGO-540)

AB = 1 for afterburner

AB ≠ 1 for no afterburner

TOWE = Engine Thrust to Weight Ratio

FDGWF = Ratio of Total Fuel on Board to Define Flight Design Gross

Weight (FDGW) where, in the Program FDGW = TOGW - (1.0 - FDGWF)

* WFUEL (Initialized at .60)

LDGWFS = Same as FDGWF, except for Structural Design Landing

Gross Weight (LDGW) where, in the Program

LDGW = TPGW - (1.0 - LDGWFS) * WFUEL - WSTOR

(Initialized at .90)

LDGWF = Same as LDGWFS, except for Landing Distance Calculations,

Using Design Landing Distance Gross Weight (LDGWLD) where,

in the Program LDGWLD = TOGW - (1.0 - LDGWF) * WFUEL - WSTOR

(Initialized at .70)

48

IPRINT = 1 for Error Checkout Messages to be Printed

= 0 for Error Checkout Messages not to be Printed

TOTHF = Thrust Modification Factor for Non-standard Day (Altitude and

Temperature Adjustment) (Initialized at 1.0)

CDF = Total C_D Modification Factor, May be Used to Adjust L/D by

Modifying C_D

CDSF = Factor to Adjust Supersonic Drag Contribution to C_{D_O}

$MISSION

N = Total Number of Mission Legs

R(I) = 0 for Warmup Fuel Allowance

= -1 to Drop Stores

= -2 to Turn at Fixed Gee

= -3 to Accelerate

= -4 for Energy Altitude Combat Fuel Allowance

= -5 for Loiter Performance

= -6 for Loiter at Fixed Altitude and Mach No.

= -7 to Turn at Maximum Possible Gee

> 0 for Climb Performance

= 1.E6 to Climb on Intermediate (Military Power)

= 1.1E6 to Climb with Afterburner with Distance Credit

GEE(I) = G Level for Turns (no 1g turns), When RANGE(I) = -7

= Time (min) on Afterburner Power, When RANGE(I) = 1.1E6

= No. of External Stores to be Dropped, When RANGE(I) = -1

NTURNS(I) = No. of Turns, When RANGE(I) = -7

= Mach, When RANGE(I) = -3

= Energy Altitude for Combat Fuel Allowance (ft), When

RANGE(I) = -4

49

TIME(I) = Loiter Time (min), When RANGE(I) = -6, or -5

= No. of Internal Stores to be Dropped, When RANGE(I) = -1

ITANK(I) = No. of External Fuel Tanks to be Dropped, When RANGE(I) = -1

AMACH(I) = Leg Mach No., Except When RANGE(I) = -1

= Weight of Cargo to be Dropped, When RANGE(I) = -1

ALT(I) = Leg Altitude

RG(I) = Distance Covered in a Particular Segment, When RANGE(I) = 1.1E6

$WEIGHTS

WAVUN = Weight of Avionics Equipment

WMA = Weight of AMMO, Guns, etc. That Does Not Require Installation

Weight in Addition

WSTORX = Weight of External Stores

WSTORI = Weight of Internal Stores

WINGF = Wing Weight Modification Factor

TAILF = Tail Weight Modification Factor

BODYF = Body Weight Modification Factor

GEARF = Gear Weight Modification Factor

$GEOM

NWOS = No. of Wing Loadings to be Cycled, Maximum of 5

WOS(I) = Wing Loadings (Max of 5) #/sq. ft.

NAR = No. of Aspect Ratios to be Cycled, Maximum of 5

AR(I) = Aspect Ratios (Max of 5)

NSWP = No. of Quarter Chord Sweep Angles to be Cycled, Maximum of 5

SWP(I) = Sweep Angles (Quarter Chord) in Degrees (Max of 5)

NTROOT = No. of Root Thickness Ratios to be Cycled, Maximum of 5

TROOT(I) = Root Thickness Ratios (Max of 5)

TAPER = Wing Taper Ratio

SFWET = Fuselage Wetted Area

SHT = Horizontal Tail Volume Coefficient

LFUS = Fuselage Length

CWRT = Root Chord of the Wing in Inches

MAC = Wing Mean Geometric Chord in Feet

SWWET = Wing Wetted Area

$PROP

IPROP = 1 for Turbofan (P&W 401)

    = 2 for Turbojet (GE J79)

    = 3 for Reciprocating (Lycoming IGO-540)

AB = 1 for afterburner

AB ≠ 1 for no afterburner

SLTH = Thrust to Weight Ratio of the Vehicle

TOWE = Thrust to Weight Ratio of the Engine

SFCF = Specific Fuel Consumption Modification Factor

To end the input and begin the mission simulation, an 'E' is placed in column 2.

The input used in this study was:

THIS IS THE FIRST TEST CASE FOR ADCA

$DESIGN

STOREX = 2., IPRINT = 0., LF = 9.75,

AMMAX = 2., VMAX = 794.,

INITER = 50.,

$END OF DESIGN

$WEIGHTS

WAVUN = 1350., WSTORX = 5500., WMA = 600.,

GEARF = .97, WINGF = .83, BODYF = .85, TAILF = .83,

$END OF WEIGHTS

$GEOM

NWOS = 5, WOSIN = 70.,80.,90.,100.,110.,

NAR = 5, ARIN = 2.5, 3.0, 3.5, 4.0, 4.5,

NTROOT = 1, TROOTIN = .04

NSWP = 1, SWPIN = 52.4, TAPER = .25, LFUS = 61.75,

$END OF GEOM

$PROP

IPROP = 1, AB = 1., SLTH = .70, TOWE = 8., SFCF = 1.2,

$END OF PROP

$MISSION

N = 11, GEE(7) = 6.5, NTURNS(7) = .8, DELM = .4, TIME(1) = .75,

    TIME(11) = 20., GMIN = 2.0,

ALT(1) = 0., 40000., 40000., 40000., 50000., 50000.,

        50000., 50000., 55000., 45000., 0.,

AMACH(1) = 0., .8, .8, .8, 1.6, 1.6, 1.6, 1.6, 1.6, .8, .4,

RANGE(1) = 0., 1.E6, 200., -3., 1.1E6, 150., -7., -1., 200., 200., -5.,

$END OF MISSION

E

Output

The values of the design variables and the corresponding performance
values were output to Tape 2 and each value was of the format F10.2.

It was also possible to have a mission summary printed out which
included an analysis of each leg in the mission along with a weight

52

summary. The component weights are also output but because of the many

approximations utilized to derive these weights, only the weight empty

and TOGW values can be considered statistically valid (Ref 4:7). This

summary data is available on Tape 6.

Appendix C

Regression Analysis Computer Program Input/Output

Input

All numeric data requires a decimal point with the exception of card number three.

Card No.	Col. No.	Description
1	1-6	The word PROGRM must appear; this is a required card
	12-17	A six character identification word in alpha-numeric format
	22-31	Number of input variables per data point including dependent variables
	32-41	Number of variables to be added by synthesis
	42-51	Number of variable synthesis cards
	52-61	Number of labeled variables
	62-71	Number of data points
2	1-6	The word PRGOPT must appear; this is a required card
	12-21	Number of Format cards if the default Format of (11X,6F10.0) is not used
	22-25	TRUE if input data is on magnetic tape otherwise leave blank
	32-35	TRUE rewinds auxiliary input data tape otherwise leave blank

| 52-55 | TRUE prints covariance and correlation matrices otherwise leave blank |
| 62-65 | TRUE calculates the equation based on a curve through the origin (zero intercept) otherwise leave blank |

| 3 | 1-3 | If variable synthesis data is on cards then the integer 0 must appear; if the data is assigned to Tape 8 then a 1 must appear; this is a required card; the number must be in column 3 |

| as needed | 1-6 | The word VARSYN must appear; these are the variable synthesis cards which form the second order terms in the equations; in general these cards are optional but were required in this study |
| | 8-10 | Synthesis operation code which can take on the following values: |

$$
\begin{aligned}
1.0 \quad & VAR(I) = Constant \\
2.0 \quad & VAR(I) = VAR(J) \\
3.0 \quad & VAR(I) = (VAR(J))^{\frac{1}{2}} \\
4.0 \quad & VAR(I) = EXP(VAR(J)) \\
5.0 \quad & VAR(I) = SIN(VAR(J)) \\
6.0 \quad & VAR(I) = COS(VAR(J)) \\
7.0 \quad & VAR(I) = TAN(VAR(J)) \\
8.0 \quad & VAR(I) = Natural\ Log\ (VAR(J)) \\
9.0 \quad & VAR(I) = ARCSIN(VAR(J)) \\
10. \quad & VAR(I) = ARCCOS(VAR(J)) \\
11. \quad & VAR(I) = VAR(J) + VAR(K) \\
12. \quad & VAR(I) = VAR(J) - VAR(K) \\
13. \quad & VAR(I) = VAR(J) * VAR(K) \\
14. \quad & VAR(I) = VAR(J)/VAR(K) \\
15. \quad & VAR(I) = VAR(J)**VAR(K)\ i.e. \\
& \qquad VAR(J)\ raised\ to\ the \\
& \qquad power = VAR(K) \\
16. \quad & VAR(I) = ARCTAN(VAR(J)/VAR(K)) \\
17. \quad & VAR(I) = (VAR(J)**2 + VAR(K)**2)^{\frac{1}{2}} \\
18. \quad & VAR(I) = MAX(VAR(J), VAR(K), Constant) \\
19. \quad & VAR(I) = MIN(VAR(J), VAR(K), Constant)
\end{aligned}
$$

	12-21	Assigns a value to I in the above operations
	22-31	Assigns a value to J in the above operations
	32-41	Assigns a value to K in the above operations
	42-61	Assigns a value to the Constant in the above operations

as needed   1-6   The word LABELS must appear; is used to assign alphanumeric titles to each of the variables, both input and synthesized; is an optional item in the program

            12-15   Index number for the first labeled variable

            16-21   A six character label for the first labeled variable

            22-25   Index number for the second labeled variable

            26-31   A six character label for the second labeled variable

            32-35   Index number for the third labeled variable

            36-41   A six character label for the third labeled variable

            . . . . .

            62-65   Index number for the sixth labeled variable

            66-71   A six character label for the sixth labeled variable

as needed   1-6   The word FORMAT must appear if the input data points are in a Format other than the default value of (11X, 6F10.0)

            12-71   Valid Format specification for the input data points; must begin with a left parenthesis and end with a right parenthesis

(Data points are input at this point)

as needed	1-6	The word PROBLM must appear; this is a required card and initiates the analysis for the selected independent variable
	12-21	The index number of the variable chosen to be the dependent variable
	22-31	Number of input or synthesized variables not allowed in the regression equation
	32-41	Limit the number of steps for calculation
	42-45	TRUE deletes the detailed printout of steps otherwise leave blank
	52-55	TRUE deletes the summary printout otherwise leave blank
	62-65	TRUE deletes the residual printout otherwise leave blank
as needed	1-6	The word DELETE must appear if this optional routine is used; is used in conjunction with the PROBLM card columns 22-31
	12-21	The index number of the first deleted variable
	22-31	The index number of the second deleted variable
	.....	
	62-71	The index number of the sixth deleted variable

(Additional PROBLM cards can be input at this time to restart the regression analysis using a new dependent variable or deleting different variables from the analysis)

Last	1-6	The word FINISH must appear; this is a required card which causes proper termination of the program

The only variable synthesis operation code used in this study was code 13., VAR(I) = VAR(J)*VAR(K). This was used to form the squared and cross product terms.

The input for this study was set up using seven input variables, the four independent design variables (VAR(1) = WOS, VAR(2) = AR, VAR(3) = TW, VAR(4) = RN) and three performance function parameters (VAR(5) = TOGW, VAR(6) = DISTL - landing distance - , VAR(7) = DISTO - takeoff distance). Regression analysis was done on all three performance functions. See Fig. for the input. Note that spacing is to be as described above.

Output

The output from the regression analysis will depend on the options selected in the input. A complete listing of the means and standard deviations for the input parameters is possible and a printout containing the covariance and correlation matrices may also be obtained. A detailed printout of the variables entered or deleted at each step of the regression analysis along with regression statistics will be received unless otherwise specified. A point by point comparison of the actual data with the calculated data will be made and yields information indicating the percent error at each point (residual information). The coefficients for the selected equation are printed and also appear as punched cards. The coefficients are available on Tape 7 with the first record having a Format of (16X, 4E16.8) and the following records have a Format of (5E16.8).

58

```
Card No.

 1    PROGRM   TEST11   7.   10.   10.   14.   45.
 2    PRGOPT   1.
 3             0
 4    VARSYN   13.  8.   1.   1.   (note: this sets VAR(8) = WOS²)
 5    VARSYN   13.  9.   1.   2.   (note: this sets VAR(9) = WOS*AR)
 .
 .
13    VARSYN   13.  17.  4.   4.   (note: this sets VAR(17) = RN²)
14    LABELS   1.00WOS    2.00AR     3.00TW     4.00RN     5.00TOGW   6.00DISTL
15    LABELS   7.00DISTO  8.00WOS**2 9.00WOS*AR 10.0WOS*TW 11.0WOS*RN 12.0AR**2
16    LABELS   13.0AR*TW  14.0AR*RN  15.0TW**2  16.0TW*RN  17.0RN**2
17    FORMAT   (10X,7F10.2)
18
 .
 .
63    45 Data Points

64    PROBLM   5.   2.   30.   TRUE   (note: this sets the dependent variable to be TOGW)
65    DELETE   6.   7.                (note: this deletes the DISTL and DISTO variables from the
                                       regression analysis)
66    PROBLM   6.   2.   30.   TRUE   (note: this sets the dependent variable to be DISTL)
67    DELETE   5.   7.                (note: this deletes TOGW and DISTO from the regression)
68    PROBLM   7.   2.   30.   TRUE   (note: this sets the dependent variable to be DISTO)
69    DELETE   5.   7.                (note: this deletes TOGW and DISTL from the regression)
```

Fig. 15. Input to the Regression Analysis

59

Appendix D

Optimization Program Input/Output

Input

Card No.	Col. No.	Description
1	1-5	The number of independent design variables (Format I5)
	6-10	The number of performance equations to be used (Format I5)
	11-15	The number of constraint equations to be used (Format I5)
as needed	1-80	The coefficients for each performance equation are input in the same format as they were output from the regression analysis. The first set of coefficients must be the function to be minimized. The second set of coefficients must be the first inequality constraint function. When all the constraints have been read in, the remaining functions are input.
as needed	1-5	Alphanumeric character for the label of the inequality constraint (Format A5)
	7-8	The inequality characters LE or GE
	10-30	The constraint value (Format G20.8)
as needed	1-80	The initial values of the design variables; provides the starting point for the search routine (Format G20.8)

Output

The output will indicate the values of the design variables and the performance functions at the minimum point. All numeric output is of the Format G16.8 and is on Tape 6.

Vita

Martin L. Marler was born 17 November 1949 in St. Louis, Missouri. He graduated from Northwest High School in St. Louis, Missouri in 1967, attended Purdue University from which he received the degree of Bachelor of Science in Aeronautical and Astronautical Engineering and a commission in the USAF in 1971. He was elected to Phi Kappa Phi while attending Purdue University. He served on a Minuteman Missile Crew at Ellsworth Air Force Base, South Dakota, from September 1971 until May 1975. During this same time period, he received the degree of Master of Arts with a major in business from the University of Northern Colorado. He entered the Air Force Institute of Technology in May 1975.

        Permanent address: 4509 Adams Drive
                             Decatur, Illinois 62526

This thesis was typed by Mrs. Frances Jarnagin.

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER GA/MC/76D-9	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A STUDY OF THE METHOD OF SURFACE FIT APPROXIMATIONS FOR AIRCRAFT OPTIMAL DESIGN		5. TYPE OF REPORT & PERIOD COVERED MS THESIS
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Martin L. Marler Captain, USAF		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Institute of Technology (AFIT-EN) Wright-Patterson AFB, Ohio 45433		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE October, 1976
		13. NUMBER OF PAGES 68
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Approved for public release; IAW AFR 190-17

JERRAL F. GUESS, Captain, USAF
Director of Information

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Aircraft Design
Aircraft Optimal Design
Optimal Design

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
A study was made to demonstrate the feasibility of using surface fit approximations in the design analysis of an advanced tactical fighter. Design variables were selected and a design space was defined based on a simple latin square method. The take-off-gross-weight (TOGW) and landing distance (DISTL) for each configuration were determined by use of a computer program which simulated the required mission. A regression analysis was performed on this data to obtain a quadratic surface fit representing TOGW and DISTL as functions of the design variables. A conjugate gradient technique was employed to find the ⟶

DD FORM 1473 1 JAN 73   EDITION OF 1 NOV 65 IS OBSOLETE

next page

Block 20. ABSTRACT (continuation)

cont

minimum TOGW subject to constraints on DISTL within the design space. This value was compared to the minimum obtained by running all possible combinations of design variables through the mission simulator. It was concluded that the surface fit approximation based on the latin square design space did yield accurate results and required fewer computer simulations.